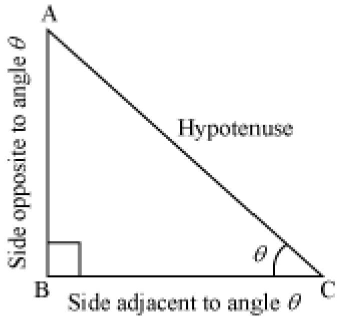
Introduction to Trigonometry

❖ Trigonometric Ratio



$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{AC}{AB}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{AC}{BC}$$

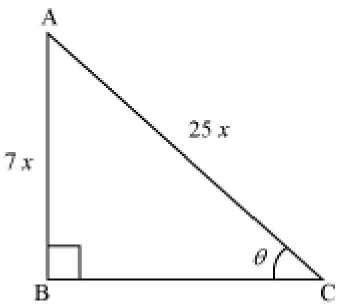
$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{BC}{AB}$$

Also, $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

If one of the trigonometric ratios of an acute angle is known, then the remaining trigonometric ratios of the angle can be calculated.

Example: If $\sin\theta = \frac{7}{25}$, then find the value of $\sec\theta(1 + \tan\theta)$.

Solution:



It is given that $\sin\theta = \frac{7}{25}$

$\sin\theta = \frac{AB}{AC} = \frac{7}{25}$

$\Rightarrow$ AB = 7x and AC = 25x, where x is some positive integer

By applying Pythagoras theorem in ΔABC, we get:

$AB^2 + BC^2 = AC^2$

$\Rightarrow (7x)^2 + BC^2 = (25x)^2$

$\Rightarrow 49x^2 + BC^2 = 625x^2$

$\Rightarrow BC^2 = 625x^2 - 49x^2$

$\Rightarrow BC = \sqrt{576}\,x = 24x$

$\therefore \sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{25}{24}$

$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{7}{24}$

$\therefore \sec\theta(1+\tan\theta) = \frac{25}{24}\left(1+\frac{7}{24}\right) = \frac{25}{24}\times\frac{31}{24} = \frac{775}{576}$

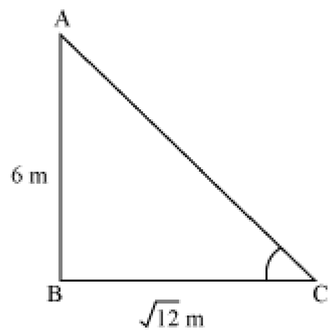
❖ **Trigonometric Ratios of some specific angles**

θ	0	30°	45°	60°	90°
sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosecθ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
secθ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cotθ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Example:

ΔABC is right-angled at B and AB = 6 m, $BC = \sqrt{12}$ m. Find the measure of ∠A and ∠C.

Solution:



$AB = 6\ \text{m},$

$BC = \sqrt{12}\ \text{m} = 2\sqrt{3}\ \text{m}$

$\tan C = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC} = \frac{6}{2\sqrt{3}} = \sqrt{3}$

$\Rightarrow \tan C = \tan 60° \qquad \left[\because \tan 60° = \sqrt{3}\right]$

$\Rightarrow \angle C = 60°$

$\therefore \angle A = 180° - (90 + 60) = 30°$

Example: Evaluate the expression

$4(\cos^3 60° - \sin^3 30°) + 3(\sin 30° - \cos 60°)$

Solution:

$4(\cos^3 60° - \sin^3 30°) + 3(\sin 30° - \cos 60°)$

$= 4\left[\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3\right] + 3\left(\frac{1}{2} - \frac{1}{2}\right)$

$= 4 \times 0 + 3 \times 0 = 0 + 0 = 0$

❖ Trigonometric Ratios of Complementary Angles

$\sin(90° - \theta) = \cos\theta \qquad \cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta \qquad \cot(90° - \theta) = \tan\theta$

$\text{cosec}(90° - \theta) = \sec\theta \qquad \sec(90° - \theta) = \text{cosec}\,\theta$

Where θ is an acute angle.

Example: Evaluate the expression: $\sin 28° \sin 30° \sin 54° \sec 36° \sec 62°$

Solution:

$$\sin 28°\sin 30°\sin 54°\sec 36°\sec 62°$$
$$=(\sin 28°\sec 62°)(\sin 54°\sec 36°)\sin 30°$$
$$=\{\sin 28°\text{cosec}(90°-62°)\}\{\sin 54°\text{cosec}(90°-36°)\}\sin 30°$$
$$=(\sin 28°\text{cosec}28°)(\sin 54°\text{cosec}54°)\sin 30°$$
$$=\left(\sin 28°\frac{1}{\sin 28°}\right)\left(\sin 54°\frac{1}{\sin 54°}\right)\times\frac{1}{2}$$
$$=\frac{1}{2}$$

Trigonometric Identities

i. $\cos^2 A+\sin^2 A=1$

ii. $1+\tan^2 A=\sec^2 A$

iii. $1+\cot^2 A=\text{cosec}^2 A$

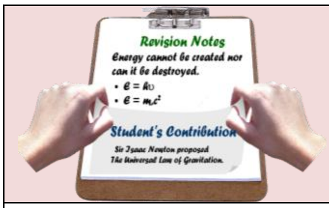
Example:

Evaluate the expression

$$4\sqrt{3}(\sin 40°\sec 30°\sec 50°)+\frac{\sin^2 34°+\sin^2 56°}{\sec^2 31°-\cot^2 59°}$$

Solution:

$$4\sqrt{3}(\sin 40°\sec 30°\sec 50°)+\frac{\sin^2 34°+\sin^2 56°}{\sec^2 31°-\cot^2 59°}$$
$$=4\sqrt{3}\left[\sec 30°(\sin 40°\sec 50°)\right]+\frac{\sin^2 34°+\sin^2(90-56°)}{\sec^2 31°-\tan^2(90-59°)}$$
$$\left[\because \cos(90°-\theta)=\sin\theta, \tan(90°-\theta)=\cot\theta\right]$$
$$=4\sqrt{3}\left[\sec 30°\sin 40°\text{cosec}(90-50°)\right]+\frac{\sin 34°+\cos^2 34°}{\sec^2 31°-\tan^2 31°}$$
$$=4\sqrt{3}\left[\frac{2}{\sqrt{3}}\sin 40°\text{cosec}40°\right]+\frac{1}{1}$$
$$=8+1=9$$



Students' Contribution to Revision Notes

Few tips to be keep in mind while proving trigonometric identities:

- ✓ Start with more complicated side of the identity and prove it equal to the other side.
- ✓ If the identity contains sine, cos and other trigonometric ratios, then express all the ratios in terms of sine and cos.
- ✓ If one side the identity can **not** be easily reduced to the other side, then simplify both the sides and then prove them as equal.
- ✓ While proving identities, **never** transfer terms from one side to the other side.

– Contributed by:

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